Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

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FURTHER MATHEMATICS

9231/32

Paper 3 Further Mechanics

October/November 2023

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10 \,\mathrm{m\,s^{-2}}$.

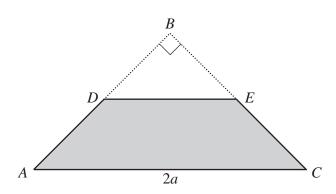
INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has 16 pages. Any blank pages are indicated.

One end of a light inextensible string of length a is attached to a fixed string is attached to a particle of mass m . The string is taut and makes a vertical through O , where $\cos \theta = \frac{2}{3}$. The particle moves in a horizontal contraction of the string is attached to a fixed string is attached to a particle of mass m . The string is taut and makes a string is attached to a particle of mass m . The particle moves in a horizontal contract of the string is attached to a fixed string is attached to a particle of mass m .	point O . The other end of the an angle θ with the downward circle with speed v .
Find v in terms of a and g .	[4

displaceme	ent from a fixed point O on the line is x m. The only forces acting on P are a 150 N \cdot	10
magnitude	$\frac{150}{(x+1)^2}$ N in the direction of increasing displacement and a resistive force of m	ıag
$\frac{450}{(11)^3}$ N.	When $t = 0$, $x = 0$ and $v = 20$.	
$(x+1)^{\circ}$	$A \dots \perp D$	
Find <i>v</i> in to determined	erms of x, giving your answer in the form $v = \frac{Ax + B}{(x+1)}$, where A and B are constant.	nts
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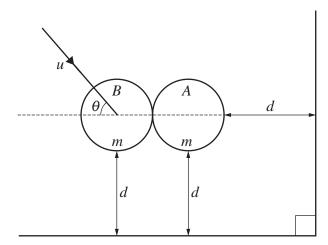


A uniform lamina is in the form of an isosceles triangle ABC in which AC = 2a and angle $ABC = 90^{\circ}$. The point D on AB is such that the ratio DB:AB = 1:k. The point E on E is such that E is parallel to E is removed from the lamina (see diagram).

mı	idpoint of AC.	
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When the lamina *ADEC* is freely suspended from the vertex *A*, the edge *AC* makes an angle θ with the downward vertical, where $\tan \theta = \frac{5}{18}$.

(b)	Find the value of k .	[3]
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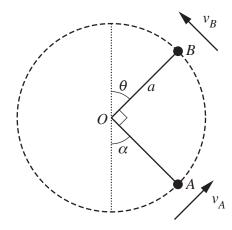


Two smooth vertical walls meet at right angles. The smooth sphere A, with mass m, is at rest on a smooth horizontal surface and is at a distance d from each wall. An identical smooth sphere B is moving on the horizontal surface with speed u at an angle θ with the line of centres when the spheres collide (see diagram). After the collision, the spheres take the same time to reach a wall. The coefficient of restitution between the spheres is $\frac{1}{2}$.

(a)	Find the value of $\tan \theta$.	[4]
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(a)



A bead of mass m moves on a smooth circular wire, with centre O and radius a, in a vertical plane. The bead has speed v_A when it is at the point A where OA makes an angle α with the downward vertical through O, and $\cos \alpha = \frac{3}{5}$. Subsequently the bead has speed v_B at the point B, where OB makes an angle θ with the upward vertical through O. Angle AOB is a right angle (see diagram). The reaction of the wire on the bead at B is in the direction OB and has magnitude equal to $\frac{1}{6}$ of the magnitude of the reaction when the bead is at A.

Find, in terms of m and g , the magnitude of the reaction at B .	[6]
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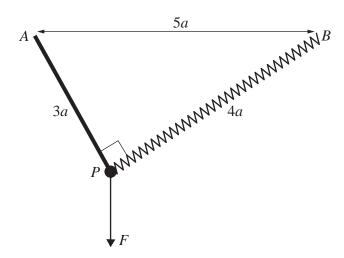
(b)	Given that $v_A = \sqrt{kag}$, find the value of k . [2]

A particle P is projected with speed u at an angle α above the horizontal from a point O on a horizontal

plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a

(a)	Derive the equation of the trajectory of <i>P</i> in the form	
	$y = x \tan \alpha - \frac{gx^2}{2u^2} \sec^2 \alpha.$	[:
	ring its flight, P must clear an obstacle of height h m that is at a horizontal opoint of projection. When $u = 40\sqrt{2} \mathrm{ms^{-1}}$, P just clears the obstacle. When	
the	ring its flight, P must clear an obstacle of height h m that is at a horizontal opoint of projection. When $u = 40\sqrt{2}\mathrm{ms^{-1}}$, P just clears the obstacle. When ieves 80% of the height required to clear the obstacle.	
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the achi	point of projection. When $u = 40\sqrt{2}\mathrm{ms^{-1}}$, P just clears the obstacle. When ieves 80% of the height required to clear the obstacle.	$u = 40 \mathrm{ms}^{-1}, P \mathrm{or}$
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A particle P of mass m is attached to one end of a light rod of length 3a. The other end of the rod is able to pivot smoothly about the fixed point A. The particle is also attached to one end of a light spring of natural length a and modulus of elasticity kmg. The other end of the spring is attached to a fixed point B. The points A and B are in a horizontal line, a distance 5a apart, and these two points and the rod are in a vertical plane.

Initially, P is held in equilibrium by a vertical force F with the stretched length of the spring equal to 4a (see diagram). The particle is released from rest in this position and has a speed of $\frac{6}{5}\sqrt{2ag}$ when the rod becomes horizontal.

(a)	Find the value of k .	[5]
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Additional page

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